basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE
NASIONALE
SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2012

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 23 pages.
Hierdie memorandum bestaan uit 23 bladsye.

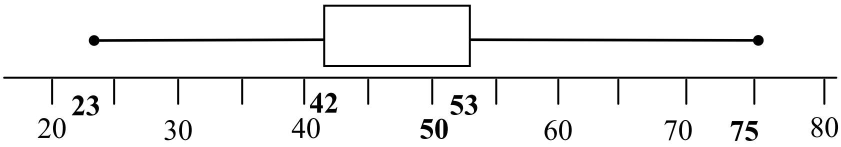
NOTE:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

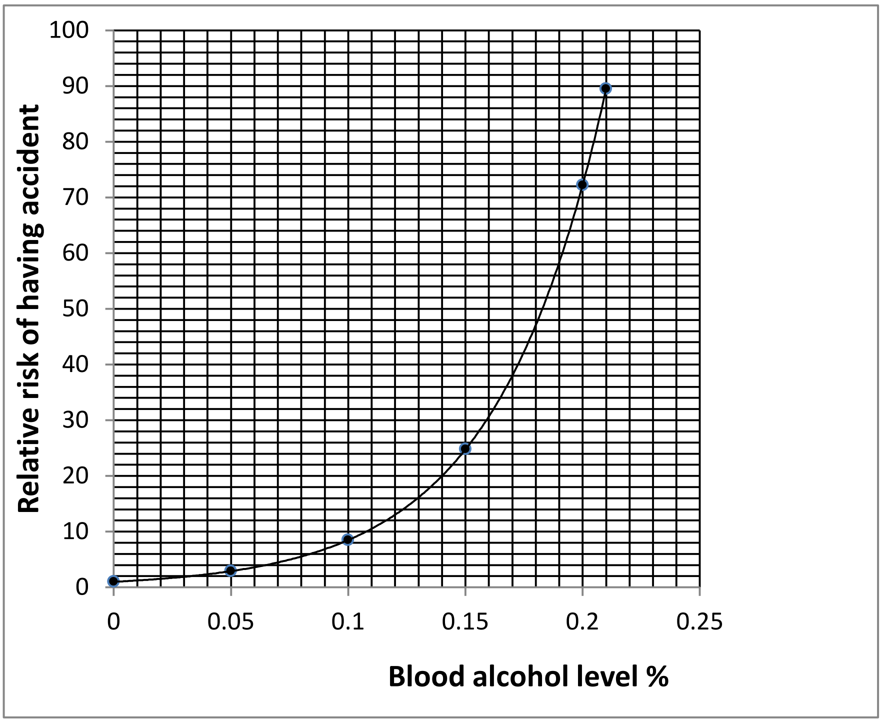
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1



1.1	Interquartile range/*Interkwartielvariasiewydte* = 53 – 42 = 11 Answer only: Full marks No CA	✓critical values (42 ; 53) ✓11 (2)
1.2	25% of trees have a height in excess of 53 cm. *25% van bome het 'n hoogte van meer as 53 cm.*	✓✓25% (2)
1.3	Between Q_2(50) and Q_3 (53)/*Tussen* Q_2 *en* Q_3 ***REASON / REDE*** The distance between these two quartiles is the smallest/*Die afstand tussen hierdie twee kwartiele is die kleinste.* **OR** The third quarter has smallest length / *Die derde kwart het die kortste lengte*	✓ Q_2 and Q_3 ✓ reason (2) **[6]**

QUESTION/*VRAAG* 2

2.1 and/en 2.2		6 points correctly plotted (3 marks) 4 points correctly plotted (2 marks) 2 points correctly plotted (1 mark) (3) ✓ exponential curve/ *eksponensiale kromme* (1)
2.3	The trend shows that as the blood alcohol levels increase, the risk of having an accident increases rapidly. *Die tendens(neiging) toon dat indien die bloed-alkoholvlakke toeneem, die risiko van 'n motorongeluk neem vinnig toe.*	✓reason (1)
2.4	Approximately 47% (Accept 44% - 51%)	✓✓ 47% (2) **[7]**

QUESTION/*VRAAG* 3

3.1	more than 15 minutes: 140 – 104 = 36 people Approximately 36 people (Accept 34 – 37) Answer only: Full marks	✓ 104 ✓ 36 (2)
3.2	At 8 minutes approximately 27 people and at 12 minutes approximately 62 people left the auditorium/*By 8 minute het ongeveer 27 mense en by 12 minute ongeveer 62 mense die ouditorium verlaat.* $\therefore 62 - 27 = 35$ Approximately 35 people left the auditorium between 8 and 12 minutes/*Ongeveer 35 mense het tussen 8 en 12 minute die ouditorium verlaat.*	✓ 27 and 62 ✓ 35 (Accept 33 – 36) (2)
3.3	Modal class/*modale klas*: $11 < x \leq 16$ **OR** $11 \leq x < 16$ Mark for critical values	✓ $11 < x \leq 16$ ✓ $11 \leq x < 16$ (1) **[5]**

QUESTION/*VRAAG* 4

	SCHOOL A	**SCHOOL B**	**SCHOOL C**
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

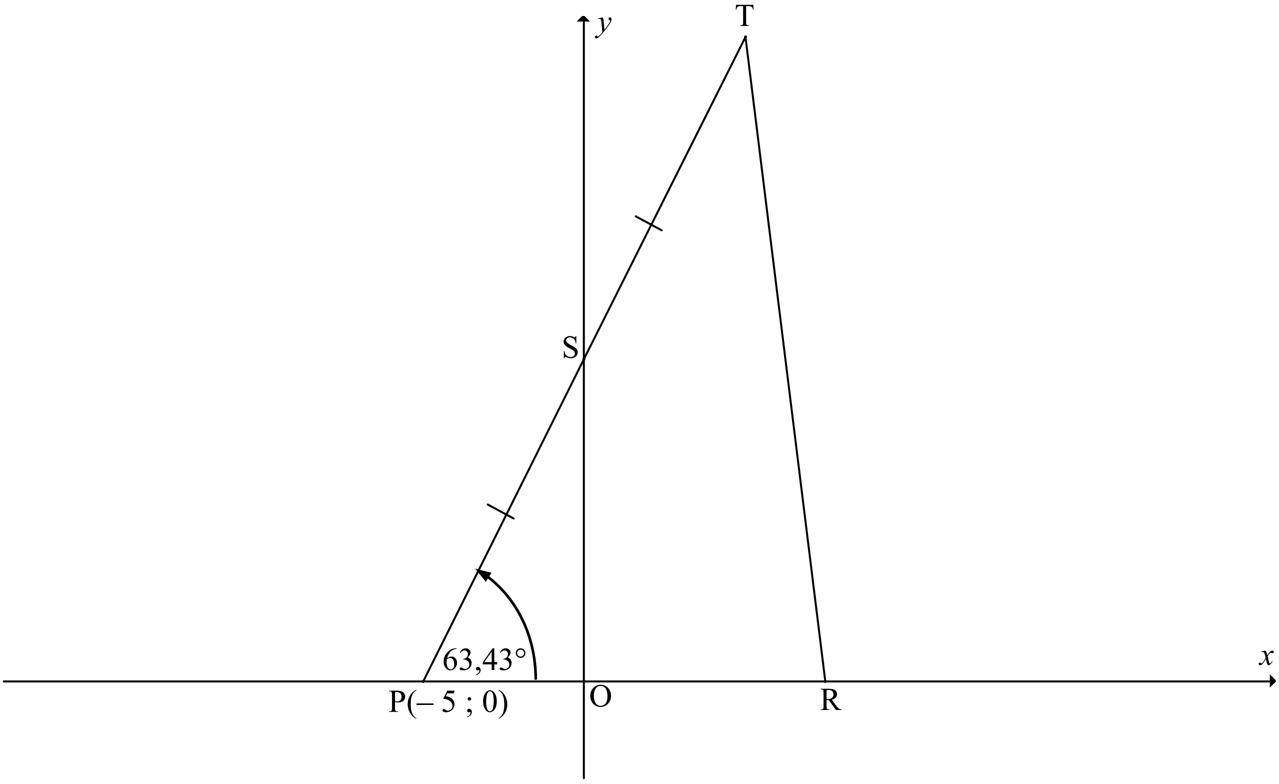
4.1	School B, because the standard deviation of B is the largest. *Skool B, want die standaardafwyking is die grootste.*	✓ School B ✓ reason (2)
4.2	There is no difference in the spread of the marks. *Daar is geen verskil in die verspreiding van punte nie.*	✓no difference / the same (1)
4.3	Add/increase each score in School A by 5 marks. *Vermeerder(tel by) elke punt in Skool A met 5 punte.*	✓ increase each mark *vermeerder elke punt* ✓ 5 marks (2)
4.4	The mean will decrease (by 10%) *Die gemiddelde sal verminder (met 10%).* The standard deviation will also decrease (by 10%) *Die standaardafwyking sal verminder (ook met 10%).*	✓ mean decreased */gemiddeld verminder* ✓ SD decreased/ *SD verminder* (2) **[7]**

Explanation why values decrease by 10%:

$$mean = \frac{\sum 0{,}9x_i}{n} = 0{,}9\frac{\sum x_i}{n} = 0{,}9\overline{x}$$

$$SD = \sqrt{\frac{\sum(x_i - \overline{x})^2}{2}} = \sqrt{\frac{\sum(0{,}9x_i - 0{,}9\overline{x})^2}{n}} = \sqrt{\frac{0{,}9^2\sum(x_i - \overline{x})^2}{n}} = 0{,}9\sqrt{\frac{\sum(x_i - \overline{x})^2}{n}}$$

QUESTION/*VRAAG* 5

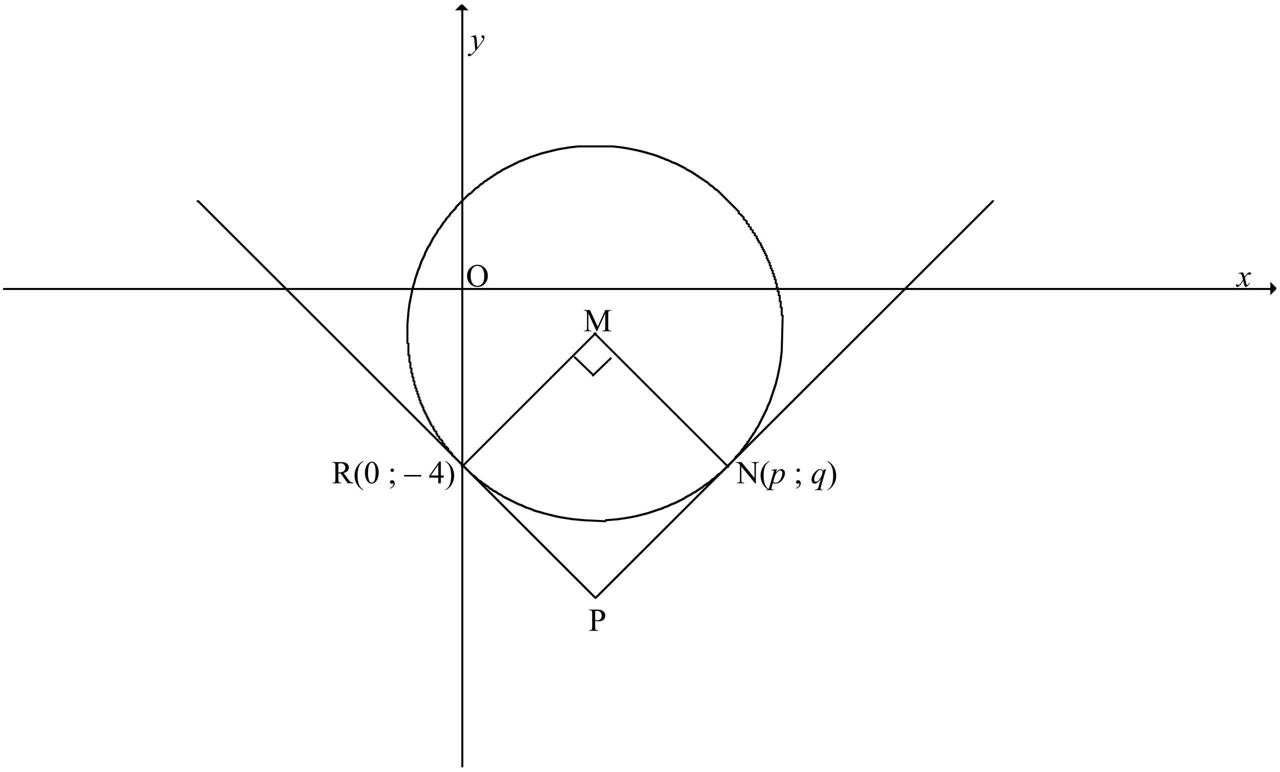


5.1.1	$m_{PT} = \tan\ 63,43°$ $= 2$	✓ tan 63,43° ✓ 2 (2) Answer only: full marks
5.1.2	Coordinates of P(– 5 ; 0) $y - y_1 = m(x - x_1)$ $y - 0 = 2(x + 5)$ $y = 2x + 10$ **OR** $y = mx + c$ $0 = (2)(-5) + c$ $c = 10$ $y = 2x + 10$ **OR** $m_{PT} = 2 = \tan 63,43°$ $\tan 63,43° = \frac{\text{OS}}{\text{OP}} = \frac{\text{OS}}{5} = 2$ $\therefore \text{OS} = 10$ $y = 2x + 10$	✓substitution of P(–5 ; 0) and *m* = 2 into equation ✓ equation (2) ✓substitution of P(–5 ; 0) and *m* = 2 into equation ✓ equation (2) ✓ $\frac{\text{OS}}{5} = 2$ ✓ equation (2)

5.1.3	$\text{OS} = 10 \text{ units}$ $\text{PS}^2 = (5)^2 + (10)^2$ $= 125$ $\text{PS} = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3)
	OR	
	P(–5 ; 0) ; OS = 10 units $\text{PS}^2 = (-5-0)^2 + (0-10)^2$ $= 25 + 100$ $= 125$ $\text{PS} = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3)
	OR	
	$\frac{PS}{5} = \frac{1}{\cos 63{,}43°}$ $\therefore PS = \frac{5}{\cos 63{,}43°}$ $PS = 11{,}18$	✓ ratio ✓ $PS = \frac{5}{\cos 63{,}43°}$ ✓11,18 (3)
	OR	
	$\frac{PS}{10} = \frac{1}{\sin 63{,}43°}$ $\therefore PS = \frac{10}{\sin 63{,}43°}$ $PS = 11{,}18$	✓ ratio ✓ $PS = \frac{10}{\sin 63{,}43°}$ ✓11,18 (3)
5.1.4	Let T be (x ; y). Then $\frac{-5+x}{2} = 0$ and $\frac{0+y}{2} = 10$ $x = 5$ $y = 20$ T(5 ; 20)	✓ 5 ✓ 20 (2)
	OR	
	by inspection: T(5 ; 20)	✓ 5 ✓ 20 (2)
5.2	$OR = \left(\frac{3}{2}\right)(5) = \frac{15}{2} = 7{,}5$ $R\left(\frac{15}{2}; 0\right)$ If only x-coordinate : 2 marks	✓$x = 7{,}5$ / $\frac{15}{2}$ ✓$y = 0$ (2)

5.3	Area $\Delta PTR = \frac{1}{2}(\text{base PR}) \times (\text{height})$ $= \frac{1}{2}(5 + \frac{15}{2}) \times 20$ $= 125$ square units	✓ area formula ✓ $5 + \frac{15}{2}$ =12,5 ✓ 20 ✓125 (4)
	OR	
	Area $\Delta PTR = \frac{1}{2} PT.PR.\sin T\hat{P}R$ $= \frac{1}{2}\left(10\sqrt{5}\right)\left(\frac{25}{2}\right)\sin 63{,}43°$ $= 124{,}99$ square units	✓ area formula ✓$10\sqrt{5}$ ✓$\frac{25}{2}$ ✓124,99 (4) **[15]**

QUESTION/*VRAAG* 6



6.1	$x^2+y^2-6x+2y-8=0$ $x^2-6x+9+y^2+2y+1=8+9+1$ $(x-3)^2+(y+1)^2=18$ $\therefore$ M(3 ; – 1) If only $(x-3)^2+(y+1)^2=r^2\ (r^2\neq 18)$, then 2 marks	✓ x^2-6x+9 ✓ y^2+2y+1 ✓ $(x-3)^2$ ✓ $(y+1)^2$ (4)
	OR	
	$x_M=-\frac{1}{2}$(coefficient of x) $x_M=-\frac{1}{2}(-6)$ $x_M=3$ $y_M=-\frac{1}{2}$(coefficient of y) $y_M=-\frac{1}{2}(2)$ $y_M=-1$ $\therefore$ M(3 ; –1)	✓ $x_M=-\frac{1}{2}(-6)$ ✓ $x_M=3$ ✓ $y_M=-\frac{1}{2}(2)$ ✓ $y_M=-1$ (4)

6.2	$m_{RM} = \frac{-1-(-4)}{3-0}$ $= 1$ y-intercept is – 4 $y = x - 4$	✓ substitution into gradient formula ✓ $m_{\text{RM}} = 1$ ✓ equation (3)
6.3	MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $\frac{q-(-1)}{p-3} = -1$ $-p+3 = q+1$ $q = 2-p$	✓✓ $m_{MN} = -1$ ✓ substitution into gradient formula ✓ $-p+3 = q+1$ (4)
	OR	
	MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $y-(-1) = -1(x-3)$ $y+1 = -x+3$ $y = -x+2$ $q = 2-p$	✓✓ $m_{MN} = -1$ ✓ $y = -x+2$ ✓ substitution into equation of line (4)
6.4	$(x-3)^2 + (y+1)^2 = 18$ $(p-3)^2 + (q+1)^2 = 18$ $(2-q-3)^2 + (q+1)^2 = 18$ $q^2 + 2q + 1 + q^2 + 2q + 1 - 18 = 0$ $2q^2 + 4q - 16 = 0$ $q^2 + 2q - 8 = 0$ $(q+4)(q-2) = 0$ $q = -4 \text{ or } q \neq 2$ $p = 6$	✓ method ✓✓$q = -4$ ✓✓$p = 6$ (5)
	OR	
	MRPN is a square/*vierkant* (rectangle with/*reghoek met* MN = MR) ∴ $\text{M}\hat{\text{P}}\text{N} = 45°$ But MR has a slope/gradient of 1, so RN | | x-axis ∴ $q = -4$ and $p = 2 - (-4) = 6$	✓ method ✓✓$q = -4$ ✓✓ $p = 6$ (5)
	OR	

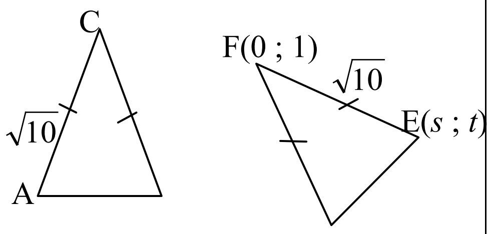
	$q = 2 - p$ $(p-3)^2 + (2-p+1)^2 = 18$ $(p-3)^2 = 9$ $\therefore \quad p - 3 = 3 \quad (p > 0)$ $p = 6$ $\therefore q = -4$ **OR** Using symmetry: $q = -4$ (since $y_M = y_R$) $-4 = 2 - p$ $p = 6$ OR $p = 2 \times 3$ (since $x_M = 2x_N$)	✓ method ✓✓ $p = 6$ ✓✓ $q = -4$ (5) ✓method ✓✓ q ✓✓ p (5)
6.5	$r^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = 52$ **OR** $p^2 + q^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = p^2 + q^2$ $= 52$	✓ substitution ✓ equation (2) ✓ substitution ✓ equation (2)
6.6	area of circle M $= \pi r^2$ $= \pi(\sqrt{18})^2$ $= 18\pi$ square *units* $= 56{,}55$*square units*	✓ $r = \sqrt{18}$ ✓ area of circle (2)
6.7	MRPN is a square (all angles equals 90°, adj sides equal) $N\hat{M}P = 45°$ (diagonals of a square bisect the angles/*hoeklyne van vierkant halveer hoeke)* $\frac{NP}{MP} = \sin N\hat{M}P$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$ **OR** MRPN is a square (all angles equals 90°, adj sides equal) $MP^2 = 18 + 18$ $= 36$ $MP = 6$ $\frac{NP}{MP} = \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ $N\hat{M}P = 45°$ ✓ ✓ $\frac{NP}{MP} = \sin N\hat{M}P$ ✓ $\frac{1}{\sqrt{2}}$ (4) ✓ $MN^2 = 18$ ✓$MP^2 = 36$ ✓6 ✓ $\frac{1}{\sqrt{2}}$ (4)

	OR	
	By inspection: P(3 ; – 7)	✓ P(3 ; – 7)
	$\frac{\text{NP}}{\text{MP}}=\frac{\sqrt{(6-3)^2+(4-7)^2}}{\sqrt{(3-3)^2+(-7+1)^2}}$ $=\frac{\sqrt{18}}{6}=\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓$\text{NP}^2 = 18$ ✓MP = 6 ✓ $\frac{1}{\sqrt{2}}$ (4) **[24]**

QUESTION/*VRAAG* 7

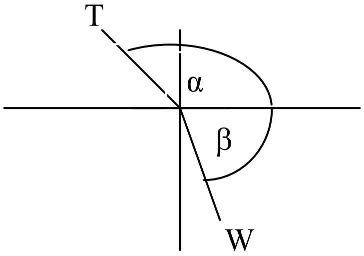
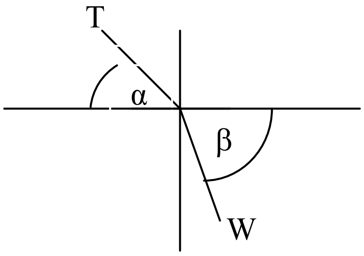
7.1	$(x \; ; \; y) \rightarrow (y \; ; \; -x) \rightarrow (-y \; ; \; -x)$	✓ $(y \; ; -x)$ ✓ $(-y \; ; -x)$ (2)
7.2	$(x \; ; y) \rightarrow (-x \; ; y) \rightarrow (y \; ; x)$	✓ $(-x \; ; \; y)$ ✓ $(y \; ; \; x)$ (2)
7.3	Mo's claim is correct/*Mo se bewering is korrek.* If order was unimportant then the image of P would be the same in both cases. This is not so./*As volgorde onbelangrik is, sal die beeld van P in beide gevalle dieselfde wees. Wat nie so is nie.* **OR** Choose any point ≠ (0 ; 0) and show that their images both cases are not the same. For example: $(3 \; ; 4) \rightarrow (4 \; ; -3) \rightarrow (-4 \; ; -3)$ $(3 \; ; 4) \rightarrow (-3 \; ; 4) \rightarrow (4 \; ; \; 3)$ Mo is correct	✓ Mo ✓ reason (2) ✓ calculation of both images ✓ Mo (2) **[6]**

QUESTION/*VRAAG* 8

8.1	ΔABC is translated by 4 units to the left and 4 units up/ *ΔABC word getransleer met 4 eenhede na links en 4 eenhede opwaarts.* Accept $(x;y) \rightarrow (x-4;y+4)$	✓ translation/ *translasie* ✓ 4 left/*links* and 4 up/*opwaarts* (2)
8.2	$R^{/} (3 \; ; -4)$	✓ 3 ✓ – 4 (2)
8.3.1	Area $\Delta A^{/}B^{/}C^{/}$ = 16 × Area of ΔABC Scale factor/*skaalfaktor* = 4	✓ 4 (1)
8.3.2	$AC = \sqrt{10}$ $A^{/}C^{/} = 4\sqrt{10}$	✓ $4\sqrt{10}$ (1)
8.4	EF = AC $\sqrt{(s-0)^2 + (t-1)^2} = \sqrt{10}$ $\sqrt{s^2 + (t-1)^2} = \sqrt{10}$ $s^2 + (t-1)^2 = 10$ $s^2 + t^2 - 2t + 1 - 10 = 0$ $s^2 + t^2 - 2t - 9 = 0$  	✓✓✓ recognising that EF = AC ✓ equation in terms of *s* and *t* (4) **[10]**

QUESTION/*VRAAG* 9

9.1	Anti-clockwise / Anti-kloksgewys: $-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $-\frac{32}{\sqrt{2}}\sin(-\theta)=8-8\sqrt{3}$ $\sin(-\theta)=\frac{-8+8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180^\circ+15^\circ$ *or* $\theta=360^\circ-15^\circ$ $=195^\circ$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5)
	OR $-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) – (2): $-\frac{32}{\sqrt{2}}\cos(-\theta)=8+8\sqrt{3}$ $\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$ $\theta=180^\circ+15^\circ$ *or* $\theta=180^\circ-15^\circ$ $=195^\circ$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ subtraction of equations ✓ value of cos θ ✓ 180° + 15° (5)
	OR Clockwise /Kloksgewys: $-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $\frac{32}{\sqrt{2}}\sin(\theta)=8-8\sqrt{3}$ $\sin(\theta)=\frac{8-8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180^\circ+15^\circ$ *or* $\theta=360^\circ-15^\circ$ $=195^\circ$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5)

	OR $-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3}$ .....(2) (1) – (2): $-\frac{32}{\sqrt{2}}\cos(\theta) = 8+8\sqrt{3}$ $\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$ $\theta=180^\circ+15^\circ$ *or* $\theta=180^\circ-15^\circ$ $=195^\circ$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ subtraction of equations ✓ value of cos θ ✓ 180° + 15° (5)
	OR  $\tan\alpha=\frac{-\frac{16}{\sqrt{2}}}{\frac{16}{\sqrt{2}}}=-1$ α = 135° $\tan\beta=\frac{-8\sqrt{3}}{8}=-\sqrt{3}$ $\beta=-60^\circ$ ∴θ = 135° + 60° = 195°	✓ tan α = – 1 ✓ 135° ✓ tan $\beta=-\sqrt{3}$ ✓ – 60° ✓ 195° (5)
	OR  tan α = 1 α = 45° tan β = $\sqrt{3}$ β = 60° ∴θ = (180° – 45°) + 60° = 195°	✓ tan α = 1 ✓ 45° ✓ tan β = $\sqrt{3}$ ✓ 60° ✓ 195° (5)

9.2	195° in 1,3 secs $\therefore$ 1 revolution/*omwenteling* in $\frac{360}{195}\times 1{,}3$ secs = 2,4 secs/*sek* 1 minute = 60 sec: $\therefore \frac{60}{2{,}4} = 25$ revolutions/*omwentelings* $\therefore$ 25 rev/min or 25 *omw/min* Answer only: 1 mark	✓ $\frac{360}{195}$ ✓ $\frac{360}{195}\times 1{,}3$ ✓ 2,4 secs ✓ $\frac{60}{2{,}4}$ ✓ 25 rev/min (5)
	OR	
	$\text{speed/sec} = \frac{195^\circ}{1{,}3} = 150^\circ/\text{sec}$ $\text{speed/minute} = 150^\circ \times 60 = 9000^\circ/\text{min}$ $\text{no of revolutions} = \frac{9000}{360} = 25\text{ rev/min}$	✓ $\frac{195}{1{,}3}$ or 150 ✓ 150×60 ✓ 9000 ✓ $\frac{9000}{360}$ ✓ 25 rev/min (5) **[10]**

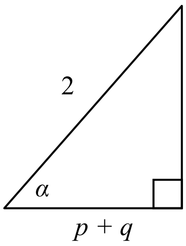
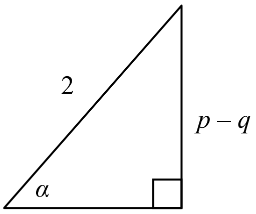
QUESTION/*VRAAG* 10

10.1	$OP^2 = 25 + 144 = 169$ $OP = 13$ $\cos\alpha = -\frac{5}{13}$ **OR** $r^2 = x^2 + y^2$ $25 + 144 = 169$ $r = 13$ $\cos\alpha = -\frac{5}{13}$ Answer only: full marks	✓ $OP^2 = 25 + 144$ ✓ *OP*= 13 ✓ answer (3) ✓ $r^2 = 25 + 144$ ✓ *r*= 13 ✓ answer (3)
10.2	$\tan(180° - \alpha)$ $= -\tan\alpha$ $= -\frac{12}{5}$	✓ − tan *α* ✓ answer (2)
10.3	$\sin(30° - \alpha)$ $= \sin 30° \cos\alpha - \cos 30° \sin\alpha$ $= \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ $= \frac{-5 + 12\sqrt{3}}{26}$	✓ expansion ✓ $\left(\frac{1}{2}\right)\left(\frac{-5}{13}\right)$ ✓ $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ (3) **[8]**

QUESTION/*VRAAG* 11

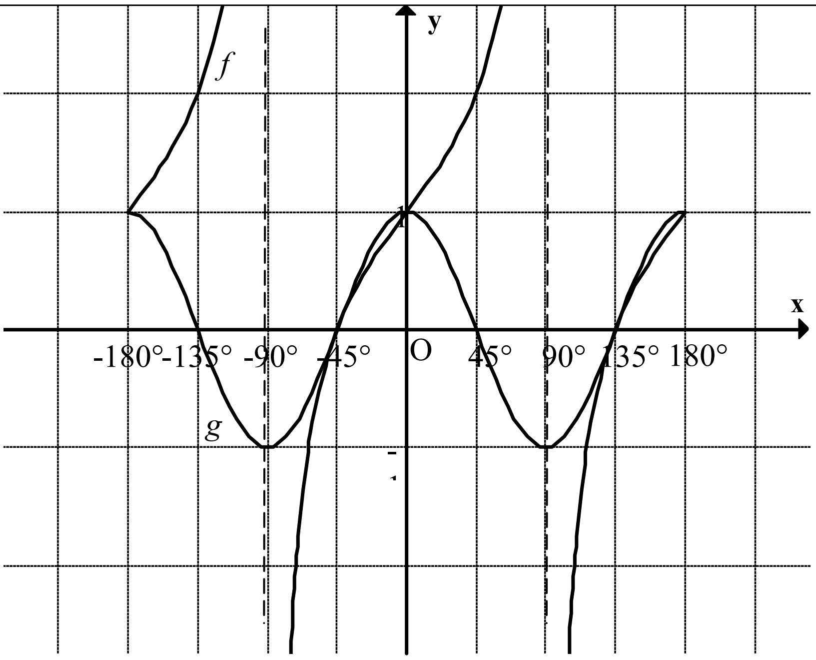
11.1	$LHS = \dfrac{\cos^2(90°+\theta)}{\cos(-\theta)+\sin(90°-\theta).\cos\theta}$ $= \dfrac{(-\sin\theta)^2}{\cos\theta+\cos\theta.\cos\theta}$ $= \dfrac{1-\cos^2\theta}{\cos\theta+\cos^2\theta}$ $= \dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta(1+\cos\theta)}$ $= \dfrac{1-\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta}-1$ $= RHS$	✓$\cos^2(90°+\theta) = \sin^2\theta$ ✓ $\sin(90°-\theta) = \cos\theta$ ✓ $\cos(-\theta) = \cos\theta$ ✓ $1-\cos^2\theta$ ✓ factors ✓$\dfrac{1-\cos\theta}{\cos\theta}$ (6)
11.2	$\tan x.\sin x + \cos x.\tan x = 0$ $\tan x(\sin x + \cos x) = 0$ $\tan x = 0$ *or* $\sin x + \cos x = 0$ $\sin x = -\cos x$ $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°;\ k \in Z$	✓ factorising ✓ tan x = 0 and sin x + cos x = 0 ✓$\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓$k.180°$ ✓ $k \in Z$ (7)
	OR	
	$\dfrac{\sin x}{\cos x}.\sin x + \cos x.\dfrac{\sin x}{\cos x} = 0$ $\dfrac{\sin^2 x}{\cos x} + \dfrac{\cos x.\sin x}{\cos x} = 0$ $\sin^2 x + \cos x.\sin x = 0$ $\sin x(\sin x + \cos x) = 0$ $\sin x + \cos x = 0$ $\sin x = 0$ *or* $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°\ ;\ k \in Z$ $(\sin x + \cos x)^2 = 0$ $1 + \sin 2x = 0$ $\sin 2x = -1$ ✓ $x = 135° + k.180°$	✓ factorising ✓ sin x = 0 and ✓$\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓$k.180°$ ✓ $k \in Z$ (7)
	OR	
	$\tan x.\sin x + \cos x.\tan x = 0$ $(\cos x \neq 0)$ $\tan^2 x + \tan x = 0$ $\tan x(\tan x + 1) = 0$ $\tan x = 0$ *or* $\tan x + 1 = 0$ $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°\ ;\ k \in Z$	✓ factorising ✓ tan x = 0 and $\tan x + 1 = 0$ ✓$\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓$k.180°$ ✓ $k \in Z$ (7)

11.3.1	$2\sin^2 3x-\sin^2 x-\cos^2 x$ $=2\sin^2 3x-(\sin^2 x+\cos^2 x)$ $=2\sin^2 3x-1$ $=-\cos 6x$ OR $2\sin^2(2x+x)-\sin^2 x-\cos^2 x$ $=2(\sin 2x.\cos x+\cos 2x.\sin x)^2-(\sin^2 x+\cos^2 x)$ $=2((2\sin x.\cos x)\cos x+(1-2\sin^2 x)\sin x)^2-1$ $=2(2\sin x.\cos^2 x+\sin x-2\sin^3 x)^2-1$ $=2(2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x)^2-1$ $=2(2\sin x-2\sin^3 x+\sin x-2\sin^3 x)^2-1$ $=2(-4\sin^3 x+3\sin x)^2-1$ $=2(16\sin^6 x-24\sin^4 x+9\sin^2 x)-1$ $=32\sin^6 x-48\sin^4 x+18\sin^2 x-1$	✓ $-(\sin^2 x+\cos^2 x)$ ✓ 1 ✓ $2\sin^2 3x-1$ (3) ✓ $-(\sin^2 x+\cos^2 x)$ ✓ 1 ✓ answer (3)
11.3.2	Max value = 1	✓ 1 (1)
11.4.1 (a)	$p=\cos\alpha+\sin\alpha$ $q=\cos\alpha-\sin\alpha$ $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $=(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)$ $=pq$ **OR** $\sin\alpha=\frac{p-q}{2}$ $\cos 2\alpha=1-2\sin^2\alpha$ $=1-2(\frac{p-q}{2})^2$ **OR** $\cos\alpha=\frac{p+q}{2}$ $\cos 2\alpha=2\cos^2\alpha-1$ $=2(\frac{p+q}{2})^2-1$ **OR** $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $=(\frac{p+q}{2})^2-(\frac{p-q}{2})^2$	✓ expansion ✓ factorise ✓ answer (3) ✓ $\frac{p-q}{2}$ ✓ expansion ✓ answer (3) ✓ $\frac{p+q}{2}$ ✓ expansion ✓ answer (3)

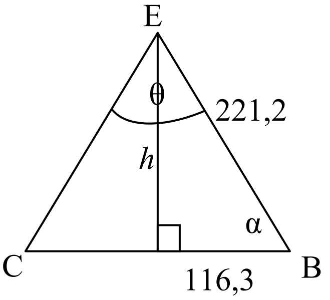
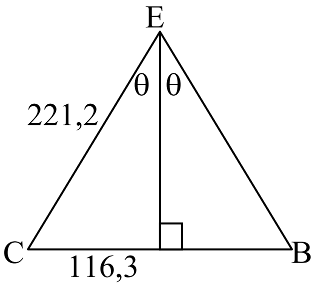
		✓ expansion ✓ $\frac{p-q}{2}$ ✓ $\frac{p+q}{2}$ (3)
11.4.1 (b)	$p+q=2\cos\alpha \quad \therefore \cos\alpha=\frac{p+q}{2}$ $p-q=2\sin\alpha \quad \therefore \sin\alpha=\frac{p-q}{2}$ $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ $=\frac{2\sin\alpha}{2\cos\alpha}$ $=\frac{p-q}{p+q}$ **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\cos\alpha=p+q$ $\cos\alpha=\frac{p+q}{2}$ $y^2=2^2-(p+q)^2$ $y=\sqrt{4-(p+q)^2}$ $\therefore \tan\alpha=\frac{\sqrt{4-(p+q)^2}}{p+q}$  **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\sin\alpha=p-q$ $\sin\alpha=\frac{p-q}{2}$  $x^2=2^2-(p-q)^2$ $x=\sqrt{4-(p-q)^2}$ $\therefore \tan\alpha=\frac{p-q}{\sqrt{4-(p-q)^2}}$ OR	✓ $p+q$ ✓ $p-q$ ✓ identity ✓ answer (4) ✓ $2\cos\alpha=p+q$ ✓ sketch ✓ $y=\sqrt{4-(p+q)^2}$ ✓ answer (4) ✓ $2\sin\alpha=p-q$ ✓ sketch ✓ $x=\sqrt{4-(p-q)^2}$ ✓ answer (4)

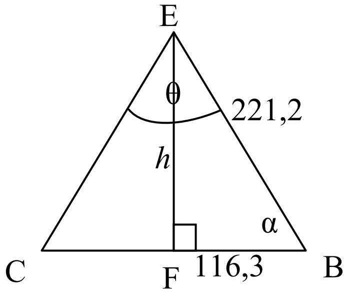
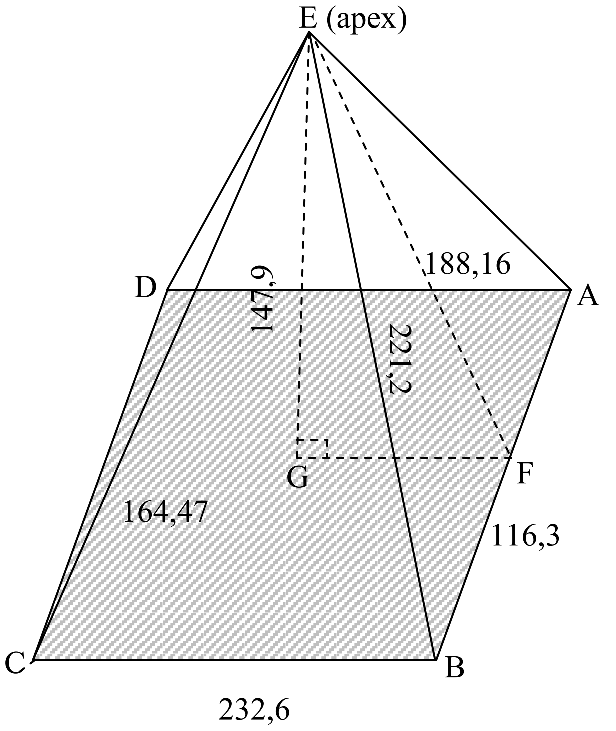
	$\cos 2\alpha = 1 - 2\sin^2\alpha$ $\sin^2\alpha = \frac{1-pq}{2}$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $\cos^2\alpha = \frac{pq+1}{2}$ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ $\tan^2\alpha = \frac{1-pq}{1+pq}$ $\therefore \tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$	✓ $\sin^2\alpha = \frac{1-pq}{2}$ ✓ $\cos^2\alpha = \frac{pq+1}{2}$ ✓ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ ✓ $\tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$ (4)
11.4.2	$\frac{p}{2q} - \frac{q}{2p}$ $= \frac{p^2-q^2}{2pq}$ $= \frac{(p+q)(p-q)}{2pq}$ $= \frac{(2\cos\alpha)(2\sin\alpha)}{2\cos 2\alpha}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$ **OR** $\frac{p}{2q} - \frac{q}{2p}$ $= \frac{\cos\alpha + \sin\alpha}{2(\cos\alpha - \sin\alpha)} - \frac{\cos\alpha - \sin\alpha}{2(\cos\alpha + \sin\alpha)}$ $= \frac{(\cos\alpha + \sin\alpha)^2 - (\cos\alpha - \sin\alpha)^2}{2(\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)}$ $= \frac{\cos^2\alpha + 2\sin\alpha\cos\alpha + \sin^2\alpha - (\cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha)}{2(\cos^2\alpha - \sin^2\alpha)}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$	✓ $\frac{p^2-q^2}{2pq}$ ✓ factorising substituting ✓ from 11.4.1(a) ✓ and 11.4.1(b) ✓ 2 sin 2α ✓ tan 2α (6) ✓ substitution ✓ single fraction ✓ 4 sinα cosα ✓ 2 cos2α ✓ 2sin2α ✓ tan2α (6) **[30]**

QUESTION/*VRAAG* 12

12.1		<u>$y = \tan x + 1$</u> ✓ asymptotes and shape for whole domain ✓ y intercept ✓ x intercepts <u>$y = \cos 2x$</u> ✓ x intercepts ✓ y intercept/TP ✓ minimum values (6)
12.2	Period of g is 180°.	✓ 180° (1)
12.3	Reflected about the x-axis and then translated by 10° to the left/ *refleksie om die x-as en dan 'n translasie van 10° links.* **OR** Translated by 10° to the left and then reflected about the x-axis/ *Translasie van 10° links en dan 'n refleksie om die x-as.*	✓ reflected about x –axis/*refleksie om x-as* ✓ 10° to the left *10° na links* (2) ✓ 10° to the left *10° na links* ✓ reflected about x –axis/*refleksie om x-as* (2)
12.4	f is always increasing $\therefore f^{/}(x) > 0$ always $\therefore g(x) > 0$ $\therefore 0° < x < 45°$ or $135° < x \leq 180°$ **OR** $\therefore x \in (0°; 45°)$ *or* $(135° ; 180°]$	✓ critical values 0° and 45° ✓ inequality ✓ critical values 135° and 180° ✓ inequality (4) **[13]**

QUESTION/*VRAAG* 13

13.1	In ΔCEB: $BC^2 = EC^2 + EB^2 - 2(EC)(EB)\cos C\hat{E}B$ $(232,6)^2 = (221,2)^2 + (221,2)^2 - 2(221,2)(221,2)\cos C\hat{E}B$ $\cos C\hat{E}B = \frac{2(221,2)^2 - (232,6)^2}{2(221,2)^2}$ $= 0,447.....$ $C\hat{E}B = 63,44°$	✓ substitution into correct formula ✓ 0,447.... ✓ 63,44° (3)
	OR $\cos\alpha = \frac{116,3}{221,2}$ $\alpha = 58,28°$ $\theta = 180° - 2(58,28°) = 63,44°$ 	✓ substitution into correct definition ✓ α = 58,28° ✓ 63,44° (3)
	OR $\sin\theta = \frac{116,3}{221,2}$ $\theta = 31,72°$ $A\hat{E}B = 2\theta = 2(31,72°) = 63,44°$ 	✓ substitution into correct formula ✓ θ = 31,72° ✓ 63,44° (3)
13.2	$EF^2 = EB^2 - BF^2$ $= (221,2)^2 - (116,3)^2$ $= 35403,75$ $EF = 188,16\,m$ $\cos\ E\hat{F}G = \frac{GF}{EF}$ $= \frac{116,3}{188,16}$ $= 0,618....$ $E\hat{F}G = 51,82°$ **OR**	✓ using Pythagoras correctly ✓ BF = 116,3 ✓ 188,16 ✓ using $\cos\ E\hat{F}G$ ✓ $\frac{116,3}{188,16}$ ✓ 51,82° (6)

	$\sin\alpha = \frac{h}{221{,}2}$ $h = 221{,}2\sin 58{,}28°$ $= 188{,}158...$ In ΔEFG : $\cos\ \text{E}\hat{\text{F}}\text{G} = \frac{116{,}3}{188{,}158...}$ $= 0{,}61809...$ $\text{E}\hat{\text{F}}\text{G} = 51{,}82°$   	✓ $\sin\alpha = \frac{h}{221{,}2}$ ✓ h subject ✓ $h = 188{,}158...$ ✓ using $\cos\ \text{E}\hat{\text{F}}\text{G}$ ✓ $\frac{116{,}3}{188{,}16}$ ✓ 51,82° (6) **[9]**

TOTAL/*TOTAAL*: 150